Draft version February 5, 2008
Preprint typeset using LaTeX style emulateapj v. 2/19/04

POLARIZATION INTERMITTENCY IN MHD AND ITS INFLUENCE ON TURBULENT CASCADE

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Draft version February 5, 2008

ABSTRACT

Goldreich-Sridhar model of incompressible turbulence provides an elegant approach to describing strong MHD turbulence. It relies on the fact that interacting Alfvénic waves are independent and have random polarization. However, in case of strong interaction a spontaneous local assymetry can arise. We used direct numerical simulations to show that polarization alignment occurs and it grows larger at smaller scales. This effect would lead to a shallower spectrum and stronger anisotropy. Even small changes in these two properties will have important astrophysical consequencies, e.g. for the cosmic ray physics.

Subject headings: turbulence: incompressible, turbulence: magnetic

1. INTRODUCTION

Astrophysical plasmas is turbulent and magnetized. Magnetohydrodynamic turbulence affects many phenomena including the propagation and acceleration of cosmic rays (see Shlickeiser 2003). While foundations of the theory of anisotropic MHD turbulence model can be traced back to works in 80s (see Shebalin, Matthaeus & Montgomery 1983, Higdon 1984, Matthaeus & Brown 1988) a substantial recent progress was initiated by the pioneering study by Goldreich & Sridhar (1995, henceforth GS95). There a concept of balancing of linear and non-linear term in incompressible MHD equations was suggested, which results in a prediction between the wavenumbers parallel $\|$ and perpendicular $\perp$ to the local direction of magnetic field $k_\| \sim k_\perp^{2/3}$. GS95 model also predicts Kolmogorov $E(k) \sim k_\perp^{-5/3}$ spectrum, which is consistent with both interstellar (see Armstrong, Rickett & Spangler 1995) and Solar wind (see Horbury 1999) data.

While the core concept of the GS95 model, namely, critical balance, was confirmed in numerical simulations (Cho & Vishniac 2000, Maron & Goldreich 2001, Cho, Lazarian & Vishniac 2002, henceforth CLV02), the simulations revealed a difference in the spectral indexes obtained. For instance, Maron & Goldreich (2001) obtained the spectral index that is close to $-3/2$ in contrast to $-5/3$ in CLV02. Index of $-3/2$ for field-perpendicular energy was obtained in simulations of Muller & Grappin (2005). This stimulated theoretical efforts to understand the difference between the theory and simulations on one hand and between different sets of simulations on the other hand. In particular, Boldyrev (2005) proposed a model in which the interactions are weakened compared to the GS95 predictions as a result of 3D structure of eddies. An alternative point of view, namely, flattening of the spectrum as the result of intermittency was discussed in Maron & Goldreich (2001).

We note, that the problem of MHD turbulence spectrum is of great practical importance. For instance, the introduction of GS95 spectrum and its extension to compressible media (see Lithwick & Goldreich 2001, Cho & Lazarian 2002, 2003) resulted in a substnatial changes in understanding of cosmic ray propagation and acceleration (see Chandran 2000, Yan & Lazarian 2002, 2004, Cho & Lazarian 2006).

In what follows we address the issue of interaction efficiency using numerical simulations. We use both compressible and incompressible MHD runs. Contrary to some earlier claims, Cho & Lazarian (2002) demonstrated weak coupling of the compressible and Alfvenic parts of the spectra, which justifies comparing Alfvenic turbulence in incompressible and compressible media.

2. INCOMPRESSIBLE MHD

In the incompressible case it is convenient to use Elsässer variables $\mathbf{w} = \mathbf{v}_A + \mathbf{v} - \mathbf{b}$, $\mathbf{u} = -\mathbf{v}_A + \mathbf{v} + \mathbf{b}$, where $\mathbf{v}_A$ and $\mathbf{b}$ are mean and total magnetic fields in velocity units (see Biskamp 2003). With those MHD equations will have a symmetric form

$$\partial_t \mathbf{u} - (\mathbf{v}_A \cdot \nabla)\mathbf{u} = -(\mathbf{w} \cdot \nabla)\mathbf{u} - \nabla P, \qquad (1)$$

$$\partial_t \mathbf{w} + (\mathbf{v}_A \cdot \nabla)\mathbf{w} = -(\mathbf{u} \cdot \nabla)\mathbf{w} - \nabla P, \qquad (2)$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{w} = 0. \qquad (3)$$

Here the total pressure $P = p + B^2/8\pi$ is determined by incompressibility conditions (3). Explicitly

$$P = \int \frac{d^3x'}{4\pi} \frac{\nabla \mathbf{w} : \nabla \mathbf{u}}{|\mathbf{x}' - \mathbf{x}|}.$$

Aside from symmetry, the remarkable property of these equations is the existence of the exact nonlinear solutions for one field, such as $\mathbf{u} = \mathbf{f}(\mathbf{r} + \mathbf{v}_A t)$, in the absence of the other, $\mathbf{w} = \mathbf{0}$.

Variables $\mathbf{u}$ and $\mathbf{w}$ can be Fourier decomposed into waves which have a dispersion relation of $\omega = v_A k_\|$. These are actually consist of two modes, shear Alfvén waves, which have $\mathbf{u}$ and $\mathbf{w}$ perpendicular to both $\mathbf{k}$ and $\mathbf{v}_A$, and pseudo-Alfvén, or slow waves that in incompressible fluid compress magnetic field. It have been shown both euristically and numerically, that in developed turbulence shear waves govern the cascade and the

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back-reaction of slow waves are relatively unimportant (see Maron & Goldreich 2001).

In equations (1) and (2), written in Fourier space, linear term could be estimated as $v_A k_\| u$ and nonlinear as $wk_\perp u$. With sufficiently small wave amplitudes linear term dominates and the theory of so called weak or wave turbulence could be built (Galtier et al, 2002). Due to the peculiar nature of the dispersion relation only waves with opposite and equal by magnitude $\mathbf{k}_\|$ can interact, to conserve momentum. Thus, only perpendicular cascade ensues, which leads to the waves with sufficiently large $k_\perp$, so that nonlinear term is no longer small. It was proposed in GS95 that from this point turbulent cascade evolves keeping approximate balance between linear and nonlinear terms, so called critical balance. The waves with larger $k_\|$ are created by decorrelation in the lateral structure ot the wave packet because of the strong interaction.

In the weak turbulence it is safely to assume that $\mathbf{u}$ and $\mathbf{w}$ wave packets have independent polarization, since they interact weakly, only once, and never meet again. In strong turbulence this is not nessesarily true as the head of the $\mathbf{u}$ wavepacket got significatly modified as it reaches the tail of the $\mathbf{w}$ wave packet. Futhermore, as we speak of the Alfvén mode, the nonlinear term is actually proportional to $wk_\perp u \sin\theta$ where θ is an angle between $\mathbf{w}$ and $\mathbf{u}$. Therefore those wave packets that have nearly parallel polarizations can survive for longer, their cascading being inhibited.

3. NUMERICAL CODES

We used several sets of turbulent data flows, produced in driven direct MHD numerical simulations. We used both incompressible pseudospectral code, which is described in detail in CLV02, and compressible isothermal ENO code, described in Cho & Lazarian 2002. In both cases we used periodic 512^3 grid, isotropic random divergentless driving with correlation time around unity in Alfvén crossing times. Three incompressible simulations had Alfvénic Mach number, calculated as the ratio of the kinetic to magnetic energy squared, $M_A = 0.7$, 1.0 and 1.4. Compressible run had $M_A = 1.0$ and sonic Mach number of around unity.

4. RESULTS

The sample spectra of the magnetic and kinetic energies are given in Fig. 1.

We are asking the question of whether polarizations of Alfvénic parts of w and z are truly independent, or they have significant correlation. Since the effect we are looking for could be small, we used two different methods of separation of scales and modes in order to assure that the effect is not due to spurious correlation which came e.g. from uncertainty in mode decomposition.

The first method was to obtain datacubes filtered in Fourier space, where only wavevectors around some $k_{\perp 0}$ and $k_{\| 0}$ were left. The uncertainty in $k_\perp$ was of the order of $k_{\perp 0}$, while $k_{\| 0}$ and its uncertainty was set according to GS95 anisotropy. We took the Alfvénic mode which has vectors perpendicular to both $\mathbf{k}$ and *mean* $\mathbf{B}$. After filtering we tranformed fields into real space. We checked that their mean square values were of the order of unity, as it has to be for the strong and local turbulence.

We than proceeded to calculate an rms value of the estimated nonlinear term $wk_\perp u|\sin\theta|$, the rms of the angular anisotropy, which is $< \sin^2\theta >^{1/2}$, both compensated by $2^{1/2}$, and the estimated nonlinear term without $\sin\theta$, or $wk_\perp u$. We refer to the ratio of the first and the third as the weakening of the interaction corresponding to wavevector k, as this is the number in which nonlinear term is smaller than a naive estimate based on independency of polarization for $\mathbf{w}$ and $\mathbf{u}$. We refer to the second term as the measure of geometrical alignment of polarization. It will be unity if w and u are not aligned.

The second method was one of a transverse structure functions (TSF), calculated with respect to the *local* magnetic field. Namely, we calculated the square root of the forth order structure function $2 < |\delta w \delta u \sin\theta|^2 >$, where the δ difference is taken between two points, connected by vector $\mathbf{l}$ perpendicular to the local magnetic field, and θ is an angle between $\delta\mathbf{w}$ and $\delta\mathbf{u}$, and similar structure functions $2 < |\sin\theta|^2 >$, and $< |\delta w \delta u|^2 >$. And again we call the square root of the second TSF the geometrical alighnment and the ratio of square roots of the first and the third as the weakening of interaction, corresponding to scale l. Here we replaced full w and z with their projections on the plane perpendicular to the $\mathbf{B}$ vector.

The above two methods have different uncertainties that go with them. The first method uses *global* mean magnetic field to do both mode decomposition and separation of scales. The second method uses *local* magnetic field for separation of scales, as we use $\mathbf{l}$ perpendicular to the local field, but it uses *approximate* method for *mode separation*, as there is some admix of the slow mode in the vectors perpendicular to B. This admix, however, has to be small on small scales as the wavevector tends to be mostly perpendicular to the magnetic field hence the vector of the slow mode is mostly along magnetic field.

If the effects of alighnment are not spurious we will see correspondence of two methods when $k \sim 1/l$. We have plotted the geometrical alighnment factor and the weakening of the interation factor for some of our datacubes on Fig. 2. As we see, there is a correspondence between two methods of calculating our factors.

We see that a purely geometric alighnment factor $2^{1/2} < \sin^2\theta >^{1/2}$ is quite close to unity. It is certainly not enough to produce significant interaction weakening. On the other hand, the "weakening factor" is sizable, being aroung 0.5 for smallest scales. This means that there is a significant correlation between wave amplitudes and the polarization alignment. In other words, polarization alignment is rather strong where there is a large turbulent field. We additionaly confirmed this by plotting the distribution of angles between Alfvén $\mathbf{w}$ and $\mathbf{u}$ for regions with different wave amplitudes. It seems that in high-amplitude regions the alignment is much stronger. Therefore we see that the effect has an intermittent nature.

5. THE MODEL

The intermittent weakening of the interaction can in principle show itself in different ways. We follow a particular model that assume, that critical balance is persistent. Indeed, if, in one region, interaction become weakened it is compensated by further growth of $k_\perp$ while $k_\|$ does not grow much because there is no significat lateral decorrelation.

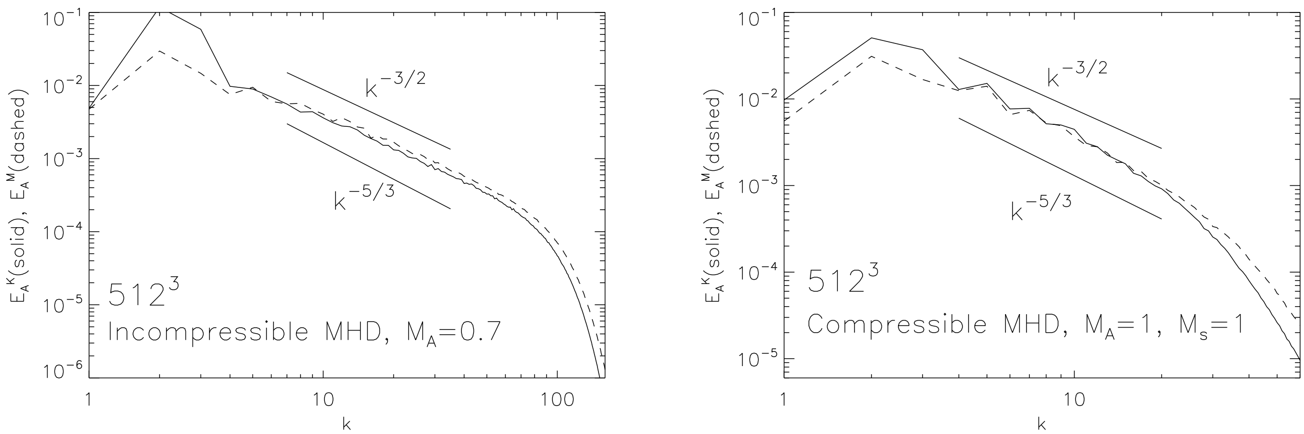

Fig. 1.— Spectra of kinetic (solid) and magnetic (dashed) energies of Alfvenic mode. *Left*: incompressible MHD, $M_A = 0.7$, hyperdiffusion of the third order, $\nu_3 \Delta^3$, *right*: compressible run with $M_A = 1$, $M_s = 1$.

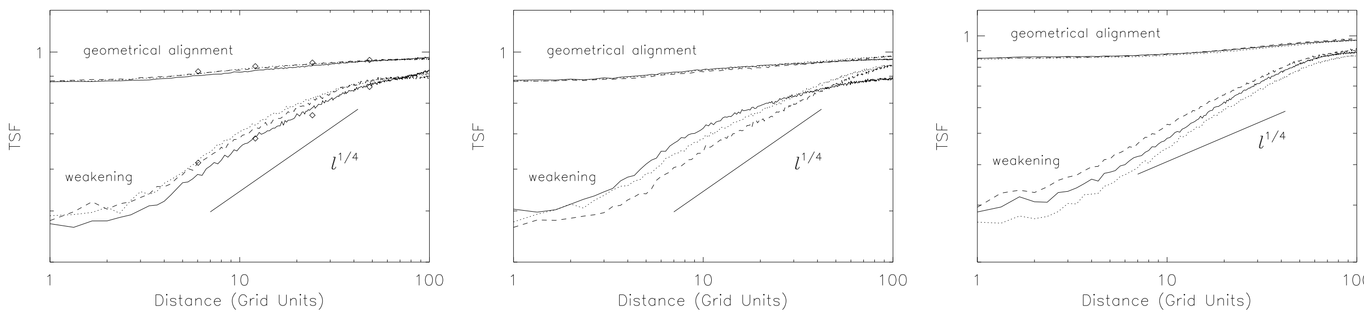

Fig. 2.— The geometrical alignment (upper curve) and the weakening of interaction factor (lower curve), *left panel*: incompressible, $M_A = 0.7$, comparison of two methods refered in text, diamonds are for the first method, they were put on distance scale $l \sim 0.7/k$, lines are for second method (TSF). Solid, dotted and dashed lines are for three simulations, separated by Alfvénic cross time. *Central panel*: incompressible, $M_A = 1.0$, same notation. *Right panel*: Compressible, $M_A = 1$, $M_s = 1$, we used a global mean magnetic field for both mode separation and scale separation.

Let's assume that weakening factor scale as $k_\perp^{-\alpha}$. Then, a critical balance would mean that $v_A \delta u k_\parallel \sim \delta u^2 k_\perp^{1-\alpha}$. Here we use u instead of both u and w, as we consider balanced case. Then cascading time could be determined by either linear or nonlinear term as $\tau \sim \delta u^{-1} k_\perp^{-1+\alpha}$. The Kolmogorov hypothethis $\delta u^2/\tau = const$ will give us the spectrum of $\delta u \sim k_\perp^{(-1+\alpha)/3}$ or $E_k \sim k_\perp^{-5/3+2\alpha/3}$. The anisotropy is obtained from critical balance: $k_\parallel \sim k_\perp^{2/3-2\alpha/3}$.

In case of $\alpha = 0$, scale independent weakening factor, we reproduce GS95 spectrum and anisotropy. In order to achieve Iroshnikov-Kraichnan spectrum we have to take $\alpha = 1/4$. However in this case anisotropy will be stronger than in GS95, namely $k_\parallel \sim k_\perp^{1/2}$.

6. INTERMITTENCY

In section 2 we chose to use 4th order structure functions (SF) to quantify interaction weakening. This choice is somewhat arbitrary. Indeed, the Kolmogorov-type arguments suggest using 3rd order functions (see Monin & Yaglom 1975), but, alas, we cannot naturally construct such a structure function in MHD (for discussion, see Biskamp 2003, sec. 7.3.3). If we assume that weakening is proportional to the wave amplitude, it is more natural to use 4th order SF. We desided to calculate the relative scaling exponents for combined Elsässer fields, as well as nonlinear term. This will quantify how much depletion of interaction or "interaction weakening" that we calculated, depends on the choice of the order of the SF.

One of the curious things we have found is that extended self-similarity (ESS, see Benzi et al., 1993) between SFs of different order is much better for nonlinear term than for combined elsasser fields. For the latter, self-similarity between SFs are mostly limited to inertial range. Obviosly there is no good extended similarity between nonlinear term and elsasser fields. We shall cautiosly conclude here, that nonlinear term, $wu \sin\theta$ is more "fundamental" than wu.

We have plotted relative scaling exponents for nonlinear term and the product of Elsässer fields on figure 3. We used 4th order SF $w^2 u^2 \sin^2\theta$ as basic, but due to ESS it is easy to recalculate them for basic SF of any order. We also plotted α, the weakening factor exponent, calculated using SFs of different order. We see that α is generally smaller than 1/4.

7. DISCUSSION

In our incompressible simulations magnetic field energy was typically slightly larger that kinetic energy (see Fig 1.) This difference is the so-called residual energy (see Muller, Grappin, 2005). Residual energy creates a

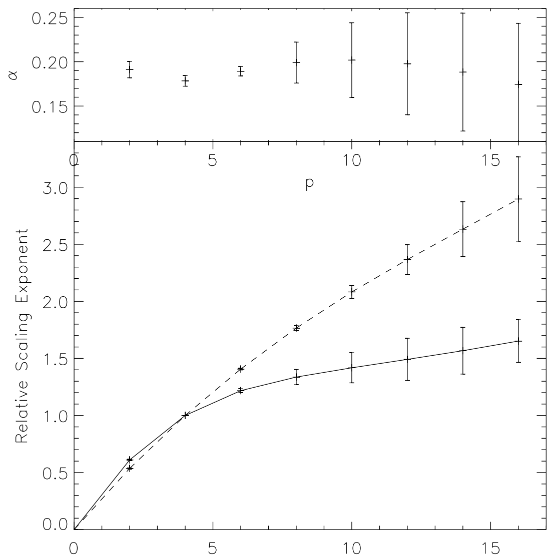

FIG. 3.— Incompressible cubes, $M_A = 0.7$, *Lower panel*: Relative scaling exponents for SFs $< |\delta w \delta u|^{p/2} >$ (solid) and $< |\delta w \delta u \sin\theta|^{p/2} >$ (dashed) with $p = 4$ taken as basic SF. *Upper panel*: interaction weakening scaling factor α (see sec. 5) calculated using different order structure functions.

polarization alignment, but of different type that we measure. In this case **w** and **u** are systematically *antiparallel*. However, we are looking not for difference between p.d.f. of angles of 0 and 180 degrees, but for difference for 0 and 90 degrees. It would be a second-order correction and is generally pretty small, as long as kinetic and magnetic energies are close. We estimated that in our case this spurious effect is never larger than 2%.

Maron and Goldreich (2001) observed axial asymmetry or net polarization of Alfvén waves. They claimed that this effect is strongest in decaying turbulence, but in forced turbulence it is bound. They did not, however, connected this effect to the flatter than $-5/3$ spectra observed in both decaying and forced runs. They speculated that spectra could be flatter due to intermittency.

Boldyrev (2005) considers weakening of interaction that comes from three-dimensional structure of the eddy. As the model he uses same Kolmogorov-type arguments, so his formulae can be reproduced by ours by taking $\alpha = \alpha'/(3 + \alpha')$, where α' is Boldyrev's α. However, in our simulations we saw that purely geometrical arguments can not fully describe weakening and the real weakening is due to correlation between angular alignment and the wave strength.

Somewhat unexpected result is that compressible cubes show more of a polarization alignment, even though they naturally have a smaller inertial range due to large numerical diffusivity and viscosity. So far within our approach α is not constrained and we do not claim that weakening will nessesarily behave as a power-law. We leave this two issues to further study.

While the magnitude of the changes of the spectral index as well as anisotropy that we observe is not large, the potential implications of this can be very substantial. Turbulence in interstellar medium is injected on the scale of dozens of parsecs (see Lazarian & Pogosyan 2000, Farmer & Goldreich 2004), and on the megaparsec scales in clusters of galaxies (see Cassano & Brunetti 2005). The scale at which Alfven turbulence dissipates can be of the order of thermal proton Larmor radius, i.e. thousands of kilometers. As the result of such a humongous scale separation any changes in the spectra and scale-dependent anisotropy will have important consequencies for the turbulent energy available at a sufficiently small scale. Cosmic ray acceleration and propagation are the processes that are directly affected.

8. SUMMARY

In the paper above we have demonstrated that

1. Turbulent magnetized flow spontaneously develop regions where the mutual shearing of the oppositely moving Alfvén waves is weakened due to polarization alignment of the waves.

2. The amplitude of waves is enhanced in the regions of correlated polarization.

3. Even though the effect of polarization alighnment is weak, it affects spectrum and anisotropy, which could have a significant impact on a wide range of astrophysical phenomena.

AL thanks Ethan Vishniac fot stimulating discussion of polarization effects during 2002 Ringberg workshop. AB thanks IceCube project for support of his research. AL acknowledges the NSF grant AST-0307869 and the support from the Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasma.

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